ON SELFINJECTIVE ALGEBRAS OF STABLE DIMENSION ZERO

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ABSTRACT

R. Rouquier has introduced a notion of a dimension of a triangulated category in [3]. One of his aims is to give a lower bound for Auslander's representation dimension: namely, he has showed that for a non-semisimple selfinjective algebra A over a field k, the representation dimension of A is greater than or equal to the dimension of the stable module category $\underline{\mathrm{mod}}\, A$ +2 (see [2]). Note that the representation dimension of A is equal to 2 if and only if A is representation-finite (due to Auslander [1]). So, if a selfinjective k-algebra A is representation-finite, then the dimension of the stable module category $\underline{\mathrm{mod}}\, A$ is equal to 0. It has been believed that the converse is also true, but this is non-trivial. We, therefore, give an argument to answer affirmatively to this in the case that k is an algebraically closed field. As consequence, if the representation dimension of a selfinjective k-algebra A is equal to 3, then the dimension of the stable module category $\underline{\mathrm{mod}}\, A$ is equal to 1.

References

[1] M. Auslander : *Representation dimension of Artin algebras*, Queen Mary College Mathematics Notes, London, 1971.
[2] R. Rouquier : *Representation dimension of exterior algebras*, Invent. Math. **165** (2006), 357-367.
[3] R. Rouquier : *Dimensions of triangulated categories*, Journal of K-theory **1** (2008), 193-256 and errata, 257-258.

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