On AS-regular algebras (joint work with Izuru Mori)

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Let k be a field.

Definition 0.1. *A connected* $\mathbb{N}$*-graded algebra* $A = k \oplus A_1 \oplus A_2 \oplus \cdots$ *is called AS-regular if it has finite global dimension* $d := \operatorname{gl.dim} \operatorname{Gr} A < \infty$ *and satisfies the following Gorenstein property:*

$$\underline{\operatorname{Ext}}^q_{\operatorname{Gr} A}(k_A, A) \cong \begin{cases} k(e) \textit{ for some } e \in \mathbb{Z} & q{=}d \\ 0 & \textit{otherwise} \end{cases}$$

The integer e *is called Gorenstein parameter.*

Remark 0.2. *In some paper these algebras are called regular algebra. In Artin-Schelter's original definition* [AS]*, (AS-)regular algebras defined by three conditions: above two conditions and finiteness of Gelfand-Kirillov dimension.*

Artin - Schelter defined AS-regular algebras to give a good class of graded algebras. Definition of AS-regular algebra extracts good homological property of polynomial algebras. Using noncommutative projective schemes and its derived category, we give a structure theorem of AS-regular algebras. This theorem shows that AS-regular algebras are polynomial algebras in some sense. We give some application of our structure theorem. We also discuss a generalization of AS-regular algebra in the case when a graded algebra is not connected over the base field k.

References

[AS] M. Artin, and W.F. Schelter, Graded algebras of global dimension 3, Adv. Math. **66** (1987), pp. 171-216.