# THICK SUBCATEGORY AND AUSLANDER CONDITION

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Through in this talk, let $R$ be a commutative Noetherian local ring, $\mathfrak{m}$ be the unique maximal ideal of $R$ and $k = R/\mathfrak{m}$ be the residue class field. We denote by $\operatorname{mod} R$ the category of finitely generated $R$-modules.

**Definition 1.** (1) For finitely generated $R$-modules $M$ and $N$, $\mathrm{P}_R(M, N)$ and $\mathrm{P}_R(M)$ are defined as follows;

$$\mathrm{P}_R(M, N) = \sup\{n \mid \operatorname{Ext}^n_R(M, N) \neq 0\}$$
$$\mathrm{P}_R(M) = \sup\{\mathrm{P}_R(M, N) \mid \mathrm{P}_R(M, N) < \infty,\ N \in \operatorname{mod} R\}$$

(2) A finitely generated $R$-module $M$ satisfies *Auslander condition (AC)* if $\mathrm{P}_R(M) < \infty$.
(3) $R$ is *AC* if there exists an integer $n$ such that $\mathrm{P}_R(M) \leq n$ for all finitely generated $R$-modules $M$.

For non-negative integer $n$, we put $\mathcal{A}_n$ the full subcategory of $\operatorname{mod} R$ consisting of all modules $M$ with $\mathrm{P}_R(M) \leq n$. It is easy to see that $R$ is AC if and only if $\mathcal{A}_n = \operatorname{mod} R$ for some $n$. We want to know that $\mathcal{A}_n$ is thick or not in general case. The following theorem is the main theorem of this talk.

**Theorem 2.** *The following conditions are equivalent.*
(1) *$R$ is AC.*
(2) *$\mathcal{A}_n = \operatorname{mod} R$ for some $n \in \mathbb{Z}_{\geq 0}$.*
(3) *$\mathcal{A}_n$ is thick for some $n \in \mathbb{Z}_{\geq 0}$.*

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