Reflection for Brauer trees

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In [1], reflection functors which introduced in [2] led to APR-tilting modules. In [3], APR-tilting modules were generalized to the following. Let Λ be a finite dimensional algebra over a field K and $P_1, \cdots, P_n$ a complete set of nonisomorphic indecomposable projective modules in mod-Λ, the category of finitely generated right Λ-modules. We set $I = \{1, \cdots n\}$. Assume that there exists a simple module $S \in$ mod-Λ satisfying $\mathrm{Hom}_\Lambda(D\Lambda, S) = 0$ and $\mathrm{Ext}^1_\Lambda(S, S) = 0$, where $D = \mathrm{Hom}_K(-, K)$. Let P_t be the projective cover of S. For $t \in I$, we set $T = \left(\bigoplus_{i \in I \setminus \{t\}} P_i\right) \oplus \tau^{-1}S$, where τ denotes the Auslander-Reiten translation. Then T is a tilting module of Λ which is called a BB-tilting module. Furthermore, in terms of derived equivalences, we know the following. We take a minimal injective presentation $0 \to S \to E^0 \xrightarrow{f} E^1$ and define a complex $E^\bullet$ as the mapping cone of the homomorphism $f : E^0 \to E^1$. Then the complex $\mathrm{Hom}^\bullet_\Lambda(D\Lambda, E^\bullet)$ is a minimal projective resolution of $\tau^{-1}S$ and hence $T^\bullet = \left(\bigoplus_{i \in I \setminus \{t\}} P_i\right) \oplus \mathrm{Hom}^\bullet_\Lambda(D\Lambda, E^\bullet)$ is a tilting complex of Λ. Now, we assume that Λ is selfinjective. Then we know that $T^\bullet$ is trivial. In this talk, we will show that if $\mathrm{Hom}_\Lambda(D\Lambda, S) \cong S$ and $\mathrm{Ext}^1_\Lambda(S, S) = 0$, then $T^\bullet = \left(\bigoplus_{i \in I \setminus \{t\}} P_i\right) \oplus \mathrm{Hom}^\bullet_\Lambda(D\Lambda, E^\bullet) \cong \left(\bigoplus_{i \in I \setminus \{t\}} P_i\right) \oplus E^\bullet$ is a tilting complex of Λ. We call the derived equivalence induced by $T^\bullet$ the reflection for Λ. Our main aim is to decide the transformations of Brauer trees given by the reflection.

References

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